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DSM2 PTM

Background

In June 1992 the Department of Water Resources hired Gilbert Bogle of Water Engineering and Modeling to develop a nonproprietary particle tracking module that DWR could adapt to output and geometry of its DSM1 model. The original module provided by Dr. Bogle was a quasi two-dimensional model which simulated longitudinal dispersion by utilizing a vertical velocity profile, vertical mixing, and a dispersion coefficient (which was a function of velocity, depth, and width of the channel). Because of the tidal nature of the water system and because of the channel grid, some complications occurred when implementing the module. After some scrutiny, the model was further developed to be a quasi three-dimensional model. Dr. Bogle continued to give further suggestions on ways to improve the module. To contact Dr. Bogle, e-mail him at gib@bogle.co.nz.

The Particle Tracking model was originally written in Fortran. Due to the nature of the model, the code was rewritten in C++ and Java to take advantage of using an object-oriented approach. Among other developments, the code was modified to handle the new output from DSM2-HYDRO. Additionally, the input system was rewritten (in Fortran) to be consistent with the DSM2-HYDRO and DSM2-QUAL input system.

Summary

The Particle Tracking Model (DSM2-PTM) simulates the transport and fate of individual notional "particles" traveling throughout the Sacramento-San Joaquin Delta. The model utilizes velocity, flow, and stage output from a one-dimensional hydrodynamic model (DSM2-HYDRO). Time intervals for these hydrodynamic values can vary but are on the order of either 15 minutes or one hour. Input into the hydrodynamic model include inflows at various rivers, exports, agricultural return and diversions, and stage at Martinez.

The Delta's geometry is modeled as a network of channel segments and open water areas connected together by junctions; the particles move throughout the network under the influence of flows and random mixing effects.

The location of a particle at any time step within a channel is given by the channel segment number, the distance from the end of the channel segment (x), the distance from the centerline of the channel (y), and the distance from the channel bottom (z) (see Figure 4.1).

Particle Movement

Three-dimensional movement of neutrally buoyant particles within channels is depicted in Figure 4.2.

Longitudinal Movement

Transverse Velocity Profile

The average cross sectional velocity during a time step, which is supplied by the hydrodynamics portion of DSM, is adjusted by multiplying it by a factor which is dependent on the particle's transverse location in the channel. This results in a transverse velocity profile where the particles located closer to the shore move slower than those located near the centerline in the channel. The model uses a quartic function to represent the velocity profile.

Vertical Velocity Profile

The average cross sectional velocity is adjusted by multiplying it by a factor which is dependent on the particle's vertical location in the channel. This results in a vertical velocity profile where the particles located closer to the bottom of the channel move slower than particles located near the surface. The model use the Von Karman logarithmic profile to represent the velocity profile.
The longitudinal distance traveled by a particle is equal to a combination of the two velocity profiles multiplied by the time step.

Transverse Movement

Transverse Mixing

Particles move across the channel due to mixing. A gaussian random factor, a transverse mixing coefficient, and the length of the time step are used in the calculation of the distance a particle will move during a time step. The mixing coefficient is a function of the water depth and the velocity in the channel. When there are high velocities and deeper water, mixing is greater.

Vertical Movement

Vertical Mixing

Particles also move vertically in the channel due to mixing. A gaussian random factor, a vertical mixing coefficient, and the length of the time step are used in the calculation of the distance a particle will move during a time step. The mixing coefficient is a function of the water depth and the velocity in the channel. As with transverse mixing, when there are high velocities and deeper water, mixing is greater.

Capabilities

- Particles can be inserted at any node location within the Delta.

- History of each Particle's movement is available. In the model, the path each particle takes through the Delta is recorded. Output useful in determining the particle's movement includes:

    - Animation. Particles are shown moving through the Delta Channels. The effects of tides, inflows, barriers, and diversions are seen at hourly time steps.

    - Number of particles passing locations. The number of particles that pass specified locations are counted at each time step.

- Each particle has a unique identity and characteristics that can change over time. Since each particle is individually tracked, characteristics can be assigned to the particle. Examples of characteristics are additional velocities that represent behavior (self-induced velocities), or the state of the particle, such as age.

- Particles travel at different velocities at different locations within the cross section. The Particle Tracking Model takes the average one-dimensional channel velocity from the DSM hydrodynamics model and creates velocity profiles from it where higher velocities occur closer to the surface of the water and towards the middle of the channel. Therefore, if particles are heavy and tend to sink towards the bottom, they will move slower than if they were neutrally buoyant. As a result, their travel time through the channels is longer.

Future Directions

Until recently, PTM simulations have primarily been made using neutrally buoyant particles. Some studies have been made where settling velocities and mortality rates have been included. These studies have concentrated on Striped Bass eggs and larvae. As additional fish data become available, additional modifications to the model will be made for future studies. These modifications will be a function of the state of the particle and the particle's environment. These modifications will require the particles react to the following:

Position

If it is known that food exists at the sides of channels, then a transverse velocity component can be included so that particles can move towards the shore.
Example: Inland Silversides may swim towards the shore for food.

Time

When particles age, their behavior may change. If eggs, their density may be different. They may sink, swim, or die.
Example: Longfin larvae are found at the surface of the water column. Juveniles are found to wards the middle of the column.

Particles may react to a diurnal cycle. An option can be included so that the particles will rise and fall depending on the time of day. This will influence their longitudinal position.

Flow

Particles can react to the tidal velocity and direction of flow.
Example: Longfin and Striped Bass move up in the water column to ride the flood tide.

Particles can have an additional longitudinal velocity component.
Example: Salmon smolts swim with the flow.

Quality

Particles' growth rate and mortality can be a function of water quality. This can include temperature, dissolved oxygen level, pesticides, and food abundance.

Particles can swim towards a certain water quality.
Example: Adult Salmon swim towards fresher water.

Theory

Movement Within a Channel

Advection within the model is represented by the one-dimensional velocity determined by DSM2-Hydro. This velocity assumes that in a cross section of the channel, the velocity is constant throughout the cross section.

Longitudinal Dispersion is caused by shear at the bottom and sides of the channel. This shear creates differences in velocities and causes turbulence within the cross section. If a tracer is injected throughout the cross section, at a distance further downstream, its concentration can be approximated by a gaussian distribution (see Figure 4.3).

This approximation is used to define the dispersion coefficient K which is one half of the change in variance with respect to time (see Figure 4.4).

Column 4 in Figure 4.5, from *Mixing in Inland and Coastal Waters* shows observed dispersion from various Rivers. (Columns 5 and 6 show theoretical and DSM1 ptm dispersion, which will be explained later.)

In order to simulate dispersion, velocity profiles and mixing are included in the model. (Figure 4.2)

The vertical velocity profile is approximated using the Von Karman Logarithmic Velocity Profile and the transverse velocity profile is approximated using a quartic function. The quartic function was chosen because it closely approximated velocity profiles measured by USGS in the Delta.

Figure 4-7a shows the movement in the x direction, the direction of the flow, that is caused by the bottom and side shear of the channel. When F_T and F_V are equal to 1, the particle is traveling at the average velocity within the channel. Aq, Bq, and Cq are currently set to 1.62, -2.22, and 0.6. Aq is used as the free parameter with Bq and Cq being derived under the assumptions that velocity is zero at channel sides and the average value of the function is 1. (Figures 4.7a, 4.7b)

Mixing and movement in the vertical and transverse directions are necessary in modeling dispersion. If only velocity profiles resulting in movement in the x direction were modeled, K would not be a constant as it is defined to be, but continually growing larger.

The mixing coefficients are described similarly to the dispersion coefficient in that they are defined as the rate of change of the variance in position. Figure 4.8 shows how the z and y distance traveled is determined. Note in the derivation, that the variance and standard deviation for both position and velocity are shown. The subscripts v and w indicate velocities and the subscripts Y and Z indicate position.

Using the gaussian random number R, a concentration distribution is created with a standard deviation of $(2E_v dt)^{1/2}$. To expand, assume that there are a large number of particles at a particular point in the cross section at the beginning of a time step. At the end of the time step, approximately 95 percent of the particles will have moved a distance equal to or less than two standard deviations or $2(2E_v dt)^{1/2}$.

The derivation for the vertical mixing coefficient Ev is shown in Figure 4.9. It is derived from the Von Karman Logarithmic Velocity Profile and is a function of depth and velocity.

The transverse mixing coefficient was determined empirically. (Figure 4.10)

Figure 4.11 shows the mixing coefficients and how the vertical and transverse distance is calculated in the model. The 0.06 and the 0.0067 can be changed in the input file.

Encountering Boundaries

When the calculated distance of travel in the vertical or horizontal is greater than the actual distance a particle can move, the particle reflects off of the boundary the additional distance that it would have moved if the boundary was not there.

For example, a channel is 10 ft .deep and the neutrally buoyant particle is located at 9.5 ft. at the beginning of a time step. The vertical mixing results in a movement of 0.7 ft. upward. The particle moves up 0.5 ft. to the 10 ft. surface and then "bounces" back 0.2 ft to the 9.8 ft. level.

To avoid excessive bouncing, smaller sub-time steps are used. The sub-time steps are calculated based on the distance traveled by particles during a time step. If the particles travel a distance larger than ten percent of the width or the depth, then the time step is reduced so that the distance traveled is equal to or less than the limiting distance. For mixing, the distance traveled is based on a gaussian distribution. Time step calculations are made for particles that travel one standard deviation away from the zero mean.

Adjustment of Position After Longitudinal Movement

After the particle has moved in the longitudinal, x direction, its position is adjusted to reflect the change in depth or width of the channel.

new z position = z(dnew/dold)

new y position = y(wnew/wold)

Z and y are the calculated positions. The "old" depth and width corresponds to the depth and width at the x position at the beginning of the time step. The "new" depth and width correspond to the depth and width at the x position at the end of the time step.

Verification

Figure 4.12 shows the derived dispersion coefficient. This calculation is not used in the model but is used as a comparison to the dispersion the model generates. To determine if the 3D formulation is adequately modeling dispersion, tests are made of the formulation using one long rectangular channel with a constant velocity. Dispersion is determined in the model by calculating the variance of the concentration of particles over time. K is checked to see if it remains relatively constant (does not increase). Model K is also compared to the derived K and the observed K shown in Figure 4.12.

(The model K—shown in Figure 4.5— was determined using a different transverse velocity profile than what is currently being used in the model.)

Checking the validity of the model in the past has also included comparing the results of the particle tracking model to results of the mass tracking model on a Delta-wide scale. Presently DSM2-PTM has been tested for bugs but has not been validated.

Movement at Junctions

When a particle reaches a junction, the decision has to be made as to where the particle is to go. Flows out of nodes include flows into channels, open water areas, agricultural diversions, and exports. Within the model, these locations are referred to as water bodies. The probability of a particle entering another water body is proportional to amount of flow entering that water body.

chance of particle entering water body = flow leaving node into water body/total flow leaving node

Movement In and Out of Open Water Areas

Once a particle enters an open water area, it no longer retains its x, y or z position. The open water area is considered fully mixed. At the beginning of a time step the volume of the open water area the volume of water leaving at each opening of the open water area is determined. From that the probability of the particle leaving the open water area is calculated.

chance of particle leaving open water = flow leaving open water area at node *time volume of open water area

Exports and Agricultural Diversions

Particles entering exports or agricultural diversions are considered "lost" from the system. Their final destination is recorded.

Movement at Ocean Boundary

Once particles pass the Martinez boundary, they have no opportunity to return to the Delta.

References

Bogle, Gilbert V. 1995. *Simulation of Dispersion in Streams by Particle Tracking*, submitted for review to J. Hydraul, ASCE

Bogle, Gilbert V. 1997. *Stream Velocity Profiles and Longitudinal Dispersion*, ASCE Jnl. of Hyd. Eng. Vol. 123 No. 9

California Dept. of Water Resources. 1994. *Methodology For Flow and Salinity Estimates in the Sacramento - San Joaquin Delta and Suisun Marsh, Fifteenth Annual Progress Report to the State Water Resources Control Board.*

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Fischer, H. B., List, E. J., Koh, R.C. Y., Imberger, J., and Brooks, N. H. 1979. *Mixing in Inland and Coastal Waters*, Academic Press., San Diego, California.

Figure 4.1 Defining Each Particle's Location

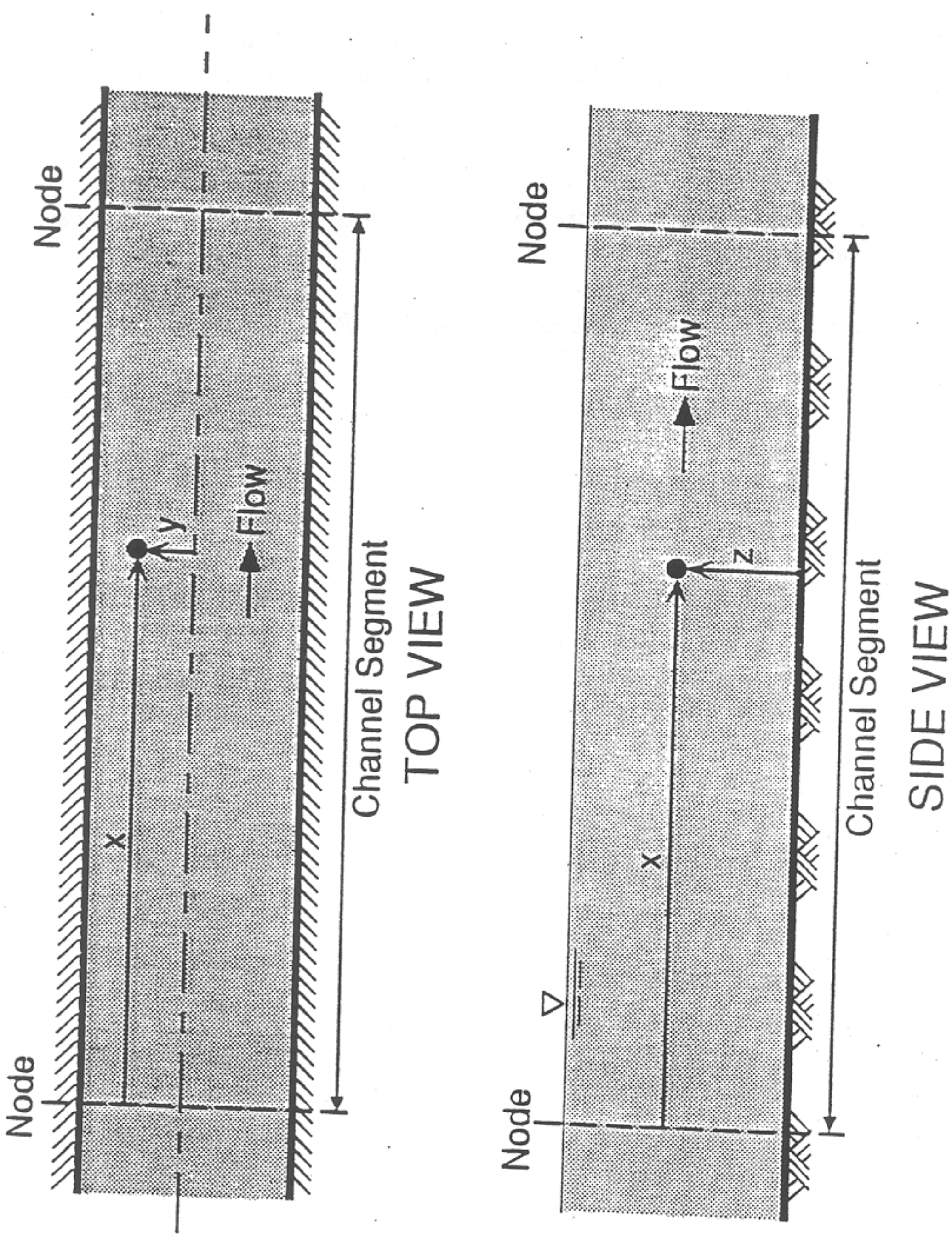


Figure 4.2 Dispersion

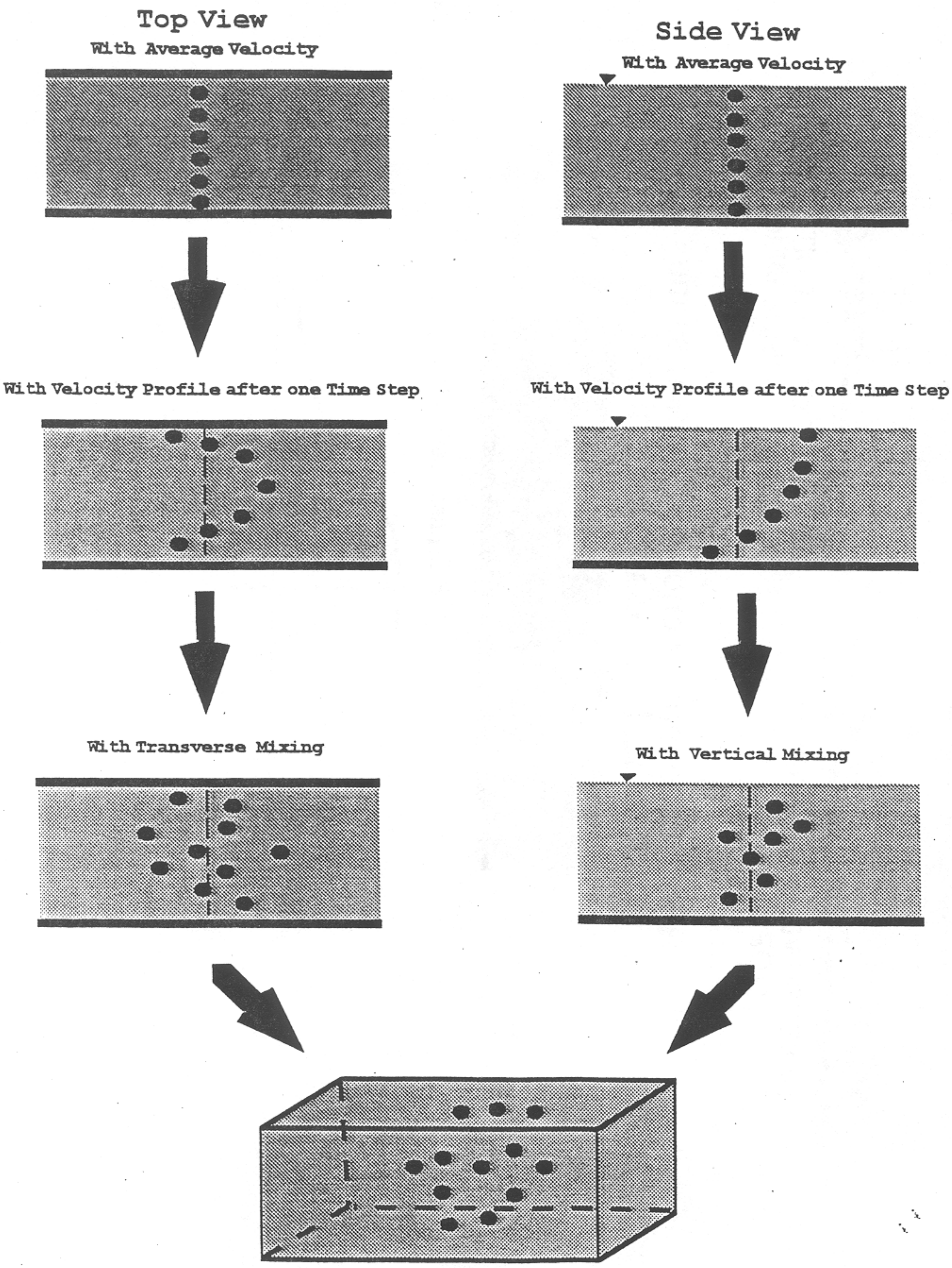


Figure 4.3 Longitudinal Dispersion

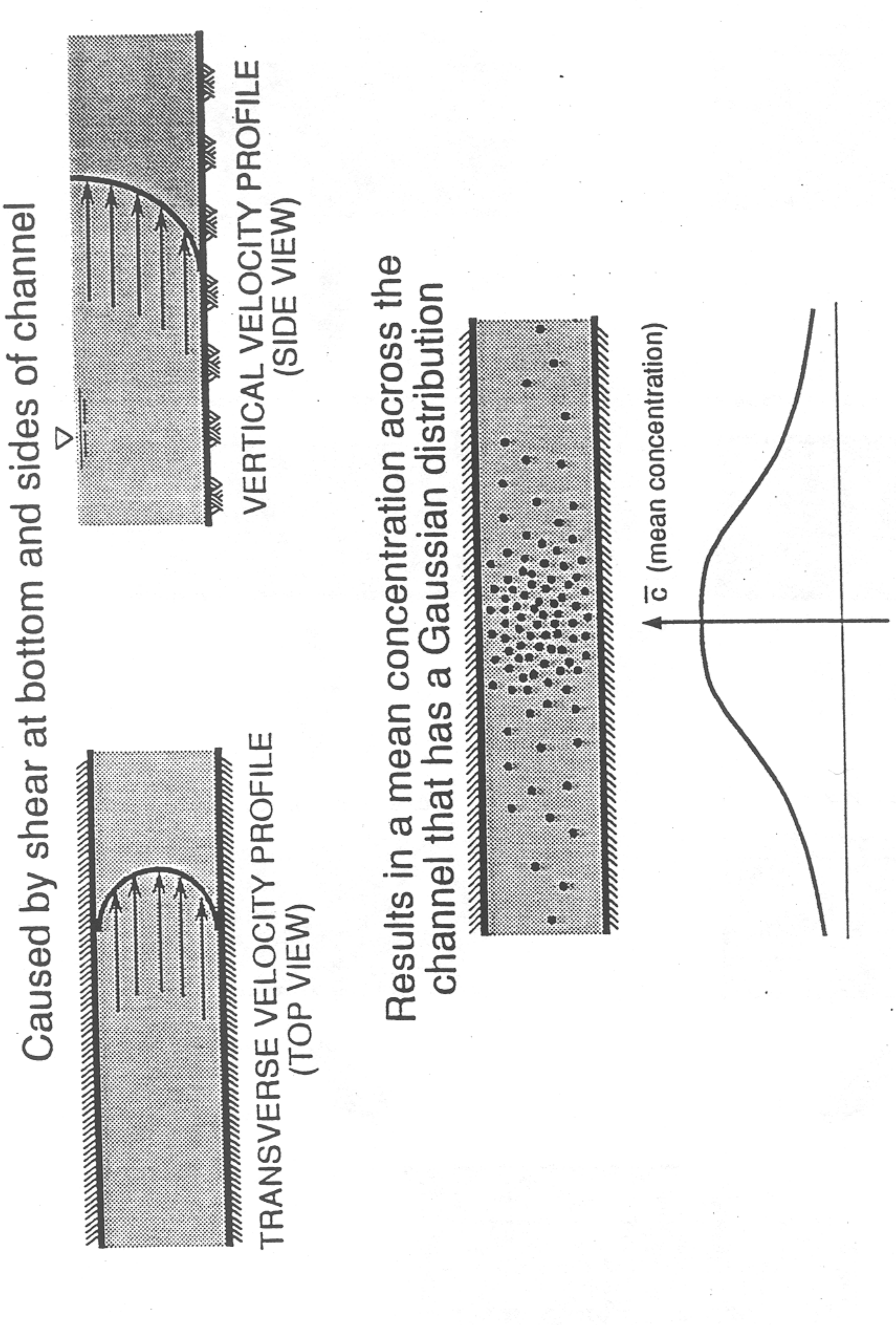
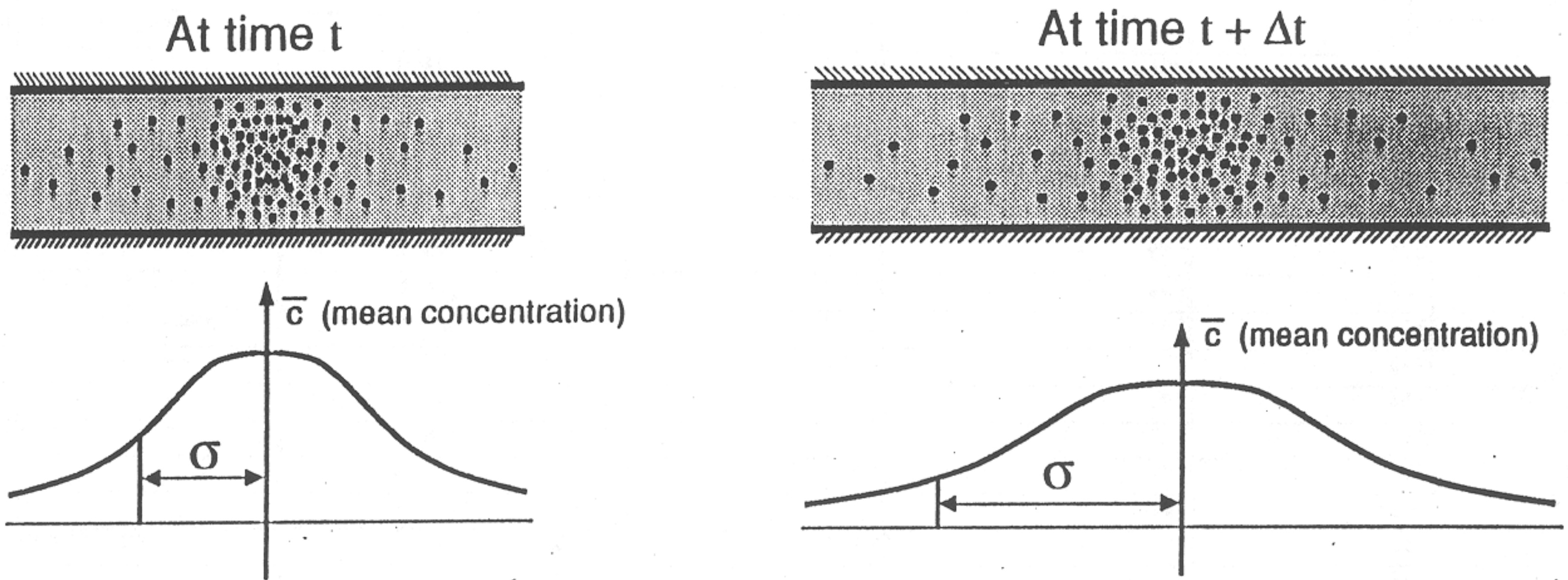


Figure 4.4 Longitudinal Dispersion (coefficient K)

The dispersion coefficient K is defined as:

$$K = \frac{1}{2}\frac{d\sigma^2}{dt}$$

where σ^2 is the variance in position with respect to the center of mass



$$K = \frac{1}{2}\left[\, \sigma^2 (t+\Delta t) - \sigma^2(t) \,\right]$$

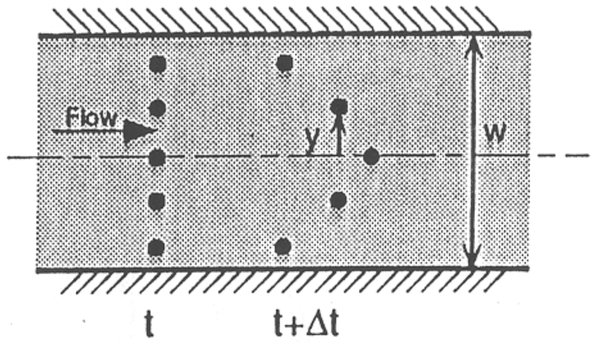
Figure 4.5 Experimental Measurements of Longitudinal Dispersion in Open Channels

Experimental Measurements of Longitudinal Dispersion in Open Channels (p.126-127 in Fischer)

	Channel	Depth (ft) (1)	Width (ft) (2)	Mean vel (ft/s) (3)	Observed K (ft^2/s) (4)	K=.11u(w^2)/d (5)	Model K (6)
1	Chicago Ship Canal	26.48	160.11	0.89	32	94	107
2	Missouri River	8.86	656.20	5.09	16153	27191	22837
3	Copper Creek	1.61	52.50	0.89	215	167	160
4		2.79	59.06	1.97	226	271	323
5		1.61	52.50	0.85	102	161	149
6	Clinch River	2.79	154.21	1.05	151	985	844
7		6.89	196.86	3.08	582	1908	1797
8		6.89	173.89	2.72	506	1315	1252
9	Copper Creek	1.31	62.34	0.52	107	171	148
10	Powell River	2.79	111.55	0.49	102	242	207
11	Clinch River	1.90	118.12	0.69	87	556	464
12	Coachella Canal	5.12	78.74	2.33	103	310	374
13	Bayou Anacoco	3.08	85.31	1.12	355	290	278
14		2.99	121.40	1.31	420	713	636
15	Nooksack River	2.49	209.98	2.20	377	4276	3684
16	Wind Bighorn River	3.61	193.58	3.24	452	3702	3366
17		7.09	226.39	5.09	1723	4046	3749
18	John Day River	1.90	82.03	3.31	151	1289	1261
19		8.10	111.55	2.69	700	454	548
20	Comite River	1.41	52.50	1.21	151	261	241
21	Sabine River	6.69	341.22	1.90	3392	3641	3090
22		15.58	416.69	2.10	7215	2573	2301
23	Yadkin River	7.71	229.67	1.41	1185	1062	962
24		12.60	236.23	2.49	2800	1215	1181

Figure 4.6 Longitudinal Movement

TRANSVERSE VELOCITY PROFILE
(TOP VIEW)

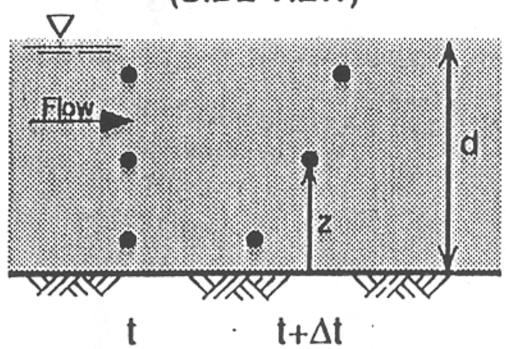


$$F_T(y) = A_q + B_q\left(\frac{2y}{w}\right)^2 + C_q\left(\frac{2y}{w}\right)^4$$

$$A_q + B_q + C_q = 0, \quad A_q + \frac{B_q}{3} + \frac{C_q}{5} = 1$$

A_q, B_q and C_q are constants

VERTICAL VELOCITY PROFILE
(SIDE VIEW)



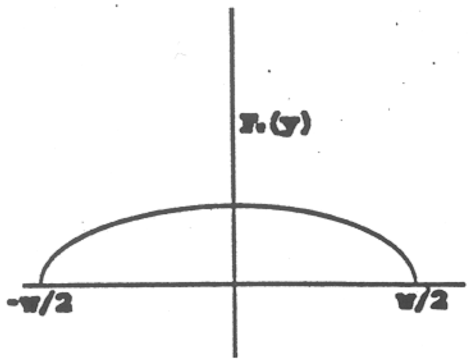
$$F_V(z) = 1 + \left(\frac{0.1}{\kappa}\right)\left[1 + \log_e\left(\frac{z}{d}\right)\right]$$

$$\kappa = 0.4$$

X Distance Traveled

$$\Delta X\,(\text{particle}) = \bar{u}_x\, F_T(y)\, F_V(z)\, \Delta t$$

Figure 4.7 DSM2-PTM Transverse Velocity Profile



$$\boxed{F_T(y) = A_q + B_q\left(2\frac{y}{w}\right)^2 + C_q\left(2\frac{y}{w}\right)^4} \qquad (1)$$

Assume that velocity is zero at sides of channel.

$$F_T\left(\frac{w}{2}\right) = 0 \qquad (2)$$

$$F_T\left(-\frac{w}{2}\right) = 0 \qquad (3)$$

Substituting eq (2) (or eq (3)) into eq (1)

$$F_T\left(\frac{w}{2}\right) = A_q + B_q\left(\frac{2(w/2)}{w}\right)^2 + C_q\left(\frac{2(w/2)}{w}\right)^4 = 0 \qquad (4)$$

$$\boxed{A_q + B_q + C_q = 0} \qquad (5)$$

Set the average value of F_T to 1.

Figure 4.7 (continued)

$$\overline{F}=\frac{\int_{-w/2}^{w/2} F_T(y)}{W}=1 \qquad (6)$$

Taking the Integral of F_T.

$$\int_{-w/2}^{w/2} A_q+B_q\left(2\frac{y}{w}\right)^2+C_q\left(2\frac{y}{w}\right)^4=A_q y+\frac{4B_q}{w^2}\frac{y^3}{3}+\frac{16C_q}{w^4}\frac{y^5}{5}\Bigg|_{-w/2}^{w/2} \qquad (7)$$

$$\int_{-w/2}^{w/2} A_q+B_q\left(2\frac{y}{w}\right)^2+C_q\left(2\frac{y}{w}\right)^4=2\left(A_q\frac{w}{2}+\frac{4B_q}{w^2}\frac{(w/2)^3}{3}+\frac{16C_q}{w^4}\frac{(w/2)^5}{5}\right)=A_q w+\frac{B_q w}{3}+\frac{C_q w}{5} \qquad (8)$$

Substituting eq (8) in eq (6).

$$\overline{F}=\frac{A_q w+\frac{B_q w}{3}+\frac{C_q w}{5}}{W}=1 \qquad (9)$$

$$\boxed{A_q+\frac{B_q}{3}+\frac{C_q}{5}=1} \qquad (10)$$

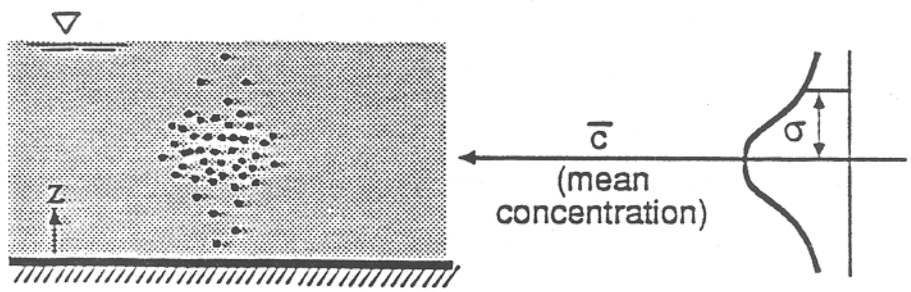
To determine the profile, use one of the coefficients as a free parameter and solve for the other two using eq (5) and eq (10). It is convenient to use Aq as the free parameter because it is equal to the centerline value.

Figure 4.8 Vertical and Transverse Displacements Due to Mixing

$$\overline{E_V} = \frac{1}{2}\frac{d\sigma_Z^2}{dt}, \quad \overline{E_T} = \frac{1}{2}\frac{d\sigma_Y^2}{dt}$$

E = mixing coefficient (vertical or transverse)

σ^2 = variance in position with respect to the center of mass



After time Δt the position is given by:

$$Y(t+\Delta t) = Y(t) + v\Delta t \qquad \text{(transverse position)}$$
$$Z(t+\Delta t) = Z(t) + w\Delta t \qquad \text{(vertical position)}$$

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v and w are transverse and vertical velocities
Y and Z are transverse and vertical positions

Since the position and velocity are statistically independent, the variance of particle position after Δt is:

$$\sigma_Y^2(t+\Delta t) = \sigma_Y^2(t) + (\sigma_v \Delta t)^2 \qquad \text{(transverse)}$$
$$\sigma_Z^2(t+\Delta t) = \sigma_Z^2(t) + (\sigma_w \Delta t)^2$$

σ_v, σ_w = standard deviation of particles' velocity

Figure 4.8 (continued)

So the change in position variance over the timestep is:

$$\Delta\sigma_Y^2 = (\sigma_V \Delta t)^2$$

$$\Delta\sigma_Z^2 = (\sigma_W \Delta t)^2$$

which results in:

$$\overline{E_T} = \frac{1}{2}\,\frac{\Delta\sigma_y^2}{\Delta t} = \frac{1}{2}\,\frac{(\sigma_V \Delta t)^2}{\Delta t}$$

$$\overline{E_V} = \frac{1}{2}\,\frac{\Delta\sigma_z^2}{\Delta t} = \frac{1}{2}\,\frac{(\sigma_W \Delta t)^2}{\Delta t}$$

rearranging the equations give:

$$\sigma_V^2 = \frac{2\overline{E_T}}{\Delta t}, \quad \sigma_W^2 = \frac{2\overline{E_V}}{\Delta t}$$

and then:

$$\sigma_V \Delta t = \sqrt{2\overline{E_T}\Delta t}$$

$$\sigma_W \Delta t = \sqrt{2\overline{E_V}\Delta t}$$

To calculate the distance traveled, a random component is introduced:

$$ydist = R\sigma_V \Delta t = R\sqrt{2\overline{E_T}\Delta t}$$

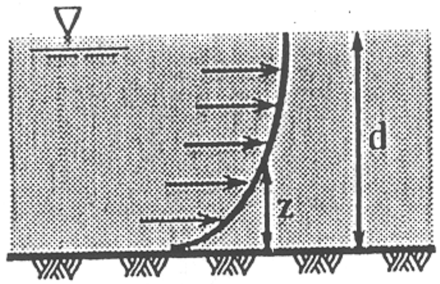
$$zdist = R\sigma_W \Delta t = R\sqrt{2\overline{E_V}\Delta t}$$

ydist, zdist = gaussian random distance traveled
R = gaussian random variable with mean 0 and variance 1 (normal distribution)

Figure 4.9 Vertical Mixing

Elder (1959) derivation for flow down an infinitely wide inclined plane

- Von Karman Logrithmic Velocity Profile



$$u' = \left(\frac{u^*}{\kappa}\right)\left(1 + \log_e \frac{z}{d}\right)$$

$u' = u - \bar{u}$
u = velocity at z location in channel
$\bar{u}$ = average velocity
κ = Von Karman constant = 0.4
$u^* = \sqrt{\frac{\tau_0}{\rho}}$ = shear velocity
τ_0 is shear stress on the bottom

- A Force Balance at any Distance from the bottom of the channel gives:

$$\tau = \rho E_V \frac{du}{dz} = \tau_0\left(1 - \frac{z}{d}\right)$$

E_V = vertical mixing coefficient

From which

$$E_V = \kappa \frac{z}{d}\left(1 - \frac{z}{d}\right) du^*$$

Averaging over the depth and taking $\kappa = 0.4$ leads to

$$\overline{E_V} = 0.067\, du^*$$

Figure 4.10 Transverse Mixing

Transverse Mixing Coefficient is determined experimentally

$$E_T = C_T\, du^*$$

E_T = Transverse mixing coefficient
d = water depth
u^* = shear velocity

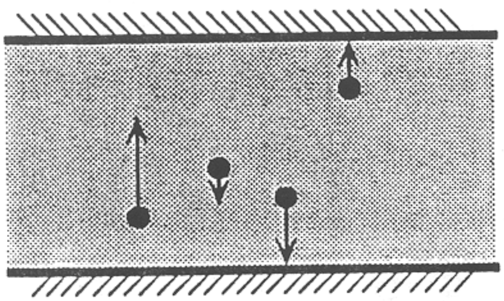
C_T = constant

for straight uniform channels
C_T is between 0.1 - 0.2

for slowly meandering streams
C_T is between 0.4 - 0.8

Figure 4.11 Mixing

TRANSVERSE MIXING
(TOP VIEW)

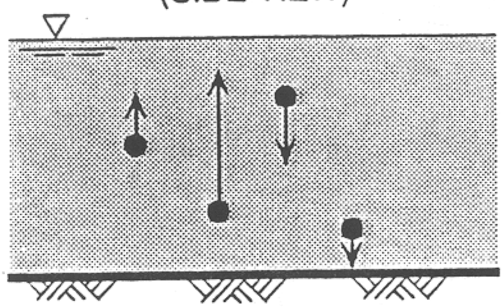


$E_T = 0.06\ d\bar{u}$

Δy_{ran} (particle) $= R\sqrt{2E_T\Delta t}$

R is a guassian random number
with mean 0 and variance 1

VERTICAL MIXING
(SIDE VIEW)



$E_V = 0.0067\ d\bar{u}^*$

Δz_{ran} (particle) $= R\sqrt{2E_V\Delta t}$

u^* = shear vel = $0.1\bar{u}$

Figure 4.12 Derived Dispersion Coefficient K

The following is a result of a derivation analogous to the molecular diffusion derivation

$$K = \frac{h^2 \overline{u'^2} I}{\overline{E_T}}$$

K = dispersion coefficient
$u' = u - \bar{u}$
u = velocity
$\bar{u}$ = mean velocity
$\overline{u'^2}$ = expected square of the deviation of the depth-averaged velocity from the mean velocity $\approx 0.2\bar{u}^2$
h = characteristic length $\approx 0.7\ W$
W = channel width
$\overline{E_T}$ = transverse mixing coefficient $\approx 0.6\ du^*$
I = a dimensionless integral of the velocity profile, approximately constant for real streams ≈ 0.07 (Bogle, 1995 suggests $I \approx 0.01$)

$$K = \frac{0.011\,\bar{u}^2 W^2}{du^*}$$

for $\overline{u'^2} = 0.2\,\bar{u}^2$
$h = 0.7\ W$
$I = 0.07$
$E_T = 0.6\ du^*$

$$u^* = \sqrt{\frac{\tau_0}{\rho}}$$

τ_0 = mean wall shear stress = $\frac{f\rho\bar{u}^2}{8}$
ρ = water density
f = Darcy-Weisbach friction factor

$u^* \approx 0.1\ \bar{u}$ for streams

$$K = \frac{0.11\ \bar{u}\ W^2}{d}$$